1. A. (10 points) Give a concise statement of Hund’s three rules.

B. (10 points) State the definition of a vector operator.

C. (10 points) If \( \mathbf{B} \) and \( \mathbf{C} \) are vector operators with respect to \( \mathbf{A} \), then what do you know about matrix elements of \( \mathbf{B} \cdot \mathbf{C} \) in the \( |AM_A\rangle \) basis?

D. (5 points) The atomic spin-orbit Hamiltonian has the form

\[ \mathbf{H}^{SO} = \sum_i \xi(r_i) \mathbf{\ell}_i \cdot \mathbf{s}_i \]

Classify \( \mathbf{H}^{SO} \) as vector or scalar with respect to \( \mathbf{J} \), \( \mathbf{L} \), and \( \mathbf{S} \). State whether \( \mathbf{H}^{SO} \) is diagonal in the \( |JM_JLS\rangle \) or \( |LM_LSM_S\rangle \) basis.

2. Consider the following multiplet transition array:

<table>
<thead>
<tr>
<th>Lower State ((L'', S''))</th>
<th>?</th>
<th>?</th>
<th>?</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>(180)</td>
<td>(16)</td>
<td>(0)</td>
</tr>
<tr>
<td>?</td>
<td>16934.63</td>
<td>48.15</td>
<td>16982.78</td>
</tr>
<tr>
<td>?</td>
<td>50.51</td>
<td>50.48</td>
<td></td>
</tr>
<tr>
<td>Upper State ((L', S'))</td>
<td>(240)</td>
<td>(16)</td>
<td></td>
</tr>
<tr>
<td>State</td>
<td>17033.29</td>
<td>64.17</td>
<td>17097.46</td>
</tr>
<tr>
<td>?</td>
<td>63.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>?</td>
<td>(310)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>?</td>
<td>17160.56</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Intensities are in parentheses above transition frequencies in cm\(^{-1}\); line separations in cm\(^{-1}\) are given between relevant transition frequencies.
A. (10 points) Use the Landé interval rule
\[ E(L, S, J) - E(L, S, J - 1) = \zeta(nLS)J \]
to determine \( J' \) and \( J'' \) values. Rather than list the \( J' \) and \( J'' \) assignments of each line, only list \( J' \) and \( J'' \) for the line observed to be most intense and for the line observed to be least intense.

B. (10 points) Use the range of \( J' \) and \( J'' \) and the intensity distribution (i.e., that the most intense transition is not \( \Delta J = 0 \)) to determine the term symbols \((2S+1)L\) for the upper and lower states. Assume \( \Delta S = 0 \).

C. (5 points) Is the upper state regular (highest \( J \) at highest term energy) or inverted (highest \( J \) at lowest term energy)? Is the lower state regular or inverted? [Partial energy level diagrams might be helpful here.]

3. A. (5 points) List the \( L-S \) terms that arise from the \((ns)(np)^2 \) and \((ns)^2(np) \) configurations. [HINT: \((np)^2 \) gives \( 1S, 3P, 1D; \) to get \( sp^2 \) couple an \( s \) electron to these three states.]

B. (5 points) Which configuration gives rise to odd terms and which to even?

C. (5 points) List the electric dipole allowed transitions between terms of the \( sp^2 \) and \( s^2p \) configurations. (Ignore fine-structure splitting of \( L-S \) terms into \( J \)-states.)

D. (10 points) Construct qualitative energy level diagrams on which you display all allowed \( J''-J' \) components of \( 2P^o-2S, 2P^o-2P, \) and \( 2P^o-2D \) transitions. Indicate which \( J''-J' \) line you would expect to be strongest for each of these three transitions.

4. (25 points) Calculate transition probabilities for the two transitions
\[
\begin{align*}
nsnp 1P_{10}^o & \rightarrow (np)^2 1S_{00} \\
nsnp 1P_{10}^o & \rightarrow (np)^2 1D_{20}
\end{align*}
\]

given the following information:
\[
\begin{align*}
1P_{10}^o & = |J = 1, M_J = 0, L = 1, S = 0 \rangle \\
& = \frac{1}{\sqrt{2}}|s0^-p0^+\rangle - \frac{1}{\sqrt{2}}|s0^+p0^-\rangle \\
1S_{00} & = |J = 0, M_J = 0, L = 0, S = 0 \rangle \\
& = \frac{1}{\sqrt{3}}|p1^-p - 1^+\rangle - \frac{1}{\sqrt{3}}|p1^+p - 1^-\rangle + \frac{1}{\sqrt{3}}|p0^+p0^-\rangle \\
1D_{20} & = \frac{1}{\sqrt{6}}|p1^+p - 1^-\rangle - \frac{1}{\sqrt{6}}|p1^-p - 1^+\rangle + \frac{2}{\sqrt{6}}|p0^+p0^-\rangle
\end{align*}
\]
The electric dipole transition moment operator, \( \mu \), does not operate on spin coordinates, is a one-electron operator, and is a vector with respect to \( \ell \). \( nsnp \rightarrow (np)^2 \) transitions are \( \Delta \ell = +1 \) processes. The relevant
\[ \Delta \ell = +1 \text{ matrix elements, as given by the Wigner-Eckart theorem for vector operators are} \]

\[
\begin{align*}
\langle n, \ell = 1, m_\ell = 1 \mid \frac{1}{2} (\mu_+ + \mu_-) \mid n, \ell = 0, m_\ell = 0 \rangle &= -\frac{1}{\sqrt{2}} \mu_+(ns) \\
\langle n, \ell = 1, m_\ell = 0 \mid \mu_+ \mid n, \ell = 0, m_\ell = 0 \rangle &= \mu_+(ns) \\
\langle n, \ell = 1, m_\ell = -1 \mid \frac{1}{2} (\mu_+ + \mu_-) \mid n, \ell = 0, m_\ell = 0 \rangle &= +\frac{1}{\sqrt{2}} \mu_+(ns)
\end{align*}
\]

where \( \mu_+(ns) \) is the reduced matrix element \( \langle np \mid \mu \mid ns \rangle \).

Show all your work including false starts. If you are unable to express the transition probabilities in terms of \( \mu_+(ns) \), lavish partial credit will be given for the ratio of transition probabilities

\[
\frac{1 P_{10}^0 - 1 S_{00}}{1 P_{10}^0 - 1 D_{00}}.
\]